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Curvaton with nonminimal derivative coupling to gravity ☆

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ABSTRACT

We show a curvaton model, in which the curvaton has a nonminimal derivative coupling to gravity. Thanks to such a coupling, we find that the scale-invariance of the perturbations can be achieved for arbitrary values of the equation-of-state of background, provided that it is nearly a constant. We also discussed about tensor perturbations, the local non-Gaussianities generated by the nonminimal derivative coupling curvaton model, as well as the adiabatic perturbations which are transferred from the field perturbations during the curvaton decay.

1. Introduction

There have been large amount of literature discussing about the curvaton mechanism [1], see also [2–5]. In the curvaton mechanism, it is assumed that the perturbations generated by inflaton itself are negligible, while the curvaton field, which is another light scalar field besides inflaton, is obliged to generate the right amount of the primordial perturbations. Although perturbations generated in this way are isocurvature ones, they can be transferred into adiabatic ones at the end of inflation, either after curvaton dominates over the inflaton, or after the curvaton decays and reaches equilibrium with the decaying products of inflaton [1]. Since the perturbations are generated by the curvaton field, the form of inflaton can be less constrained, and large non-Gaussianities are also possible. See [6–13] about related works.

In the simplest curvaton case, the curvaton field is just a canonical field with negligible mass and interactions. However, it can be extended to more complicated models with arbitrary forms of Lagrangian or coupling terms. In this Letter, we study a kind of curvaton with its kinetic term nonminimally coupled to the gravity. As will be explained in the next section, this kind of coupling has very salient feature of giving rise to scale-invariant power spectrum without knowing the evolution behavior of the universe. Moreover, similarly to the Galileon models, this model can also get rid of "ghost modes", even if the nonminimal coupling may violate the Null Energy Condition. We give full analysis of this model, especially on the perturbations. We study how it generates scale-invariant power spectrum for scalar perturbations, as well as what the tensor perturbations will be like. The stabilities of these perturbations give further constraints on this model. We also study how the field perturbations can be transferred into curvature perturbations, as well as the local-type non-Gaussianities generated by this model.

This Letter is organized as follows: in Section 2 we generally summarize our motivation of having nonminimal derivative coupling in the curvaton model. In Section 3, we introduce the nonminimal derivative coupling curvaton, and briefly review its evolution behavior in background level. Section 4 and Section 5 are devoted to the perturbation generated by the curvaton model. In Section 4, we analyze the linear scalar perturbation and its power spectrum. In the first subsection, we see that the power spectrum is indeed scale-invariant as we expected, although gravitational perturbations are considered for a comprehensive study, and small tilt of the spectrum can be given by the corrections from the potential term of the curvaton. In the second subsection we consider the tensor perturbations of our model. We find that the tensor perturbation gives more tight constraints on our model, and that a healthy tensor perturbation in our model can hardly be deviated from that of a minimal coupling single scalar model. In Section 5 we discuss about how the field perturbations can be transferred into adiabatic ones through the decay of curvaton or equilibrium with the background of the universe, and we also derive the local non-Gaussianity generated by the curvaton in this section. We make our final conclusion in Section 6.

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2. Our motivation

In general, the curvaton is simply acted by a canonical scalar field which decouples from other matters and minimally couples to gravity. The action of the curvaton field φ is described as

$$\mathcal{S}_\varphi = \int d^4x\sqrt{-g}\,P(X,\varphi), \tag{1}$$

where P is an arbitrary function of φ and its kinetic term: $X \equiv -\nabla_\mu\varphi\nabla^\mu\varphi/2$.[1] The simplest case is that φ is a canonical field, thus $P(X,\varphi)$ reduces to the form of $X - V(\varphi)$. Setting $\varphi \to \varphi_0 + \delta\varphi$, one can get the perturbed action of curvaton as:

$$\delta S_\varphi = \int d^3x\,d\eta\,\frac{a^2 Q}{c_s^2}\left[\delta\varphi'^2 - c_s^2(\partial\delta\varphi)^2\right], \tag{2}$$

where

$$Q \equiv P_{,X}, \qquad c_s^2 \equiv \frac{P_{,X}}{\rho_{,X}}, \tag{3}$$

and $'$ means derivative with respect to conformal time η. Here we have assumed its effective mass is negligible. Current observational data favors the scale-invariance property of primordial perturbations [14]. As can be derived from (2), the scale-invariance of the perturbations generated by curvaton requires [15]

$$z^2 \equiv \frac{a^2 Q}{c_s^2} \sim \frac{1}{(\eta_* - \eta)^2} \quad \text{or} \quad (\eta_* - \eta)^4, \tag{4}$$

where the first case is for that the dominant mode of perturbations is a constant one, while the second is for that the dominant mode being an increasing one.

For the canonical field case, one has $Q = 1$. Provided the universe evolves with a constant EoS w, one roughly has $a \sim -[H(\eta_* - \eta)]^{-1}$, thus from Eq. (4), it can only be that $H \sim$ constant, which is inflation ($w \simeq -1$) for the first case, or $H \sim \eta^3$, which is the matter-like contraction ($w \simeq 0$) for the second case. Here we also neglect the variation of c_s^2. Therefore we see that, the perturbations generated by curvaton are scale invariant only for limited cases where few values of w can be chosen, which we think is too tight a constraint on the evolution behavior of the early universe.

Can we have a model of which Eq. (4) is satisfied automatically, independent of how the universe evolves, whether expands or contracts, and whatever w is? If the answer is yes, then we can have much wider possibilities of the early universe evolution, which can allow much more fruitful interesting cosmological phenomena. To realize this, basically we need either Q or c_s^2, or both, change with time. For example, if we still treat c_s^2 as a constant and only Q to be time-varying, Eq. (4) requires Q scale as:

$$Q \sim \frac{1}{a^2(\eta_* - \eta)^2} \quad \text{or} \quad (\eta_* - \eta)^4/a^2 \tag{5}$$

for constant/increasing-mode-dominating case, respectively.

There have been some possibilities of time-varying Q presented in the literature, in which the curvaton field becomes non-canonical. The popular example recently proposed is to have curvaton generate scale-invariant perturbations via the so-called "conformal mechanism" [16–18], which has been applied on Galileon-genesis [19] (see also slow expansion scenario for different case [20]) or Galileon bounce models [11]. Clever idea as it is though, in these models Q is written as functions of background field ϕ, therefore it depends severely on the details of the background evolution. In general, this will need more or less tuning of the background field and become less controllable. The aim of this Letter is to look for a kind of model of which the variation of Q is universal, giving rise to required behavior (5) without worrying about the background evolution.

As has been mentioned before, the relation $a \sim -[H(\eta_* - \eta)]^{-1}$ holds universally for arbitrary evolution, provided that the EoS w is a constant. Therefore, the first relation of (5) indicates that Q should be proportional to H^2. Since $Q \sim P_{,X}$, a natural conjecture of the kinetic term of the lagrangian (1) can be RX or $G_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi$, where R is the Ricci scalar and $G_{\mu\nu}$ is the Einstein tensor, respectively. Both of the two terms appear very commonly in the literature as "nonminimal derivative coupling" [21]. In homogeneous and isotropic FRW background, both of the two terms will be reduced to $H^2\dot\phi^2$, although the first one has a correction from time-derivative of H. From this property, we expect that models with such a kinetic term can automatically satisfy the relation (5), without knowing exactly the detailed background evolution of the universe. This is our very motivation of studying this kind of curvaton model in this Letter.

3. The model

The action of nonminimal derivative coupling curvaton is considered as

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{g}\left[\frac{R}{16\pi G} + \frac{\xi}{M^2}G_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi - V(\varphi) + \mathcal{L}_{bg}\right], \tag{6}$$

where $G_{\mu\nu}$ is the Einstein tensor: $G_{\mu\nu} \equiv R_{\mu\nu} - g_{\mu\nu}R/2$, and ξ is an arbitrary coefficient.

Being first proposed by Amendola in 1992 [21] where the most general terms were given, nonminimal derivative coupling has been applied to various aspects of cosmology, for example, see [22–24] for inflation (see [25] for the reheating process), see [26–31] for dark energy, see [32] for bouncing cosmology, and see [33–35] for the study of black hole physics making use of nonminimal derivative couplings. In [36] (see also [37]), it was pointed out that such a term only leads to second order field equations and showed the exact cosmological solutions, in [38] Daniel and Caldwell analyzed the (in)stabilities of and put constraints on such kind of model, and in [39], Gao showed that nonminimal derivative coupling field can act not only as dark energy, but also as dark matter. Moreover, when coupled to several Einstein tensors, it can also give rise to inflationary behavior.

The nonminimal derivative coupling can also be viewed as a subset of Galileon models [40,41]. One of the appealing properties of this kind of model is that, while the action contains higher derivative of the field or nonminimal coupling, due to the delicate design of the lagrangian, the equation of motion of the field remains of second order, which can be free of ghost. The simplest Galileon model, which contains the coupling term ($X\Box\varphi$), can be applied to curvaton scenario, where the higher derivative term can give rise to local non-Gaussianities of $\mathcal{O}(10)$ [42]. In this Letter we try to apply another kind of Galileon model on the curvaton scenario.

From action (6), one can straightforwardly obtain that the energy density and pressure are expressed as

$$\rho_\varphi = \frac{9\xi}{M^2}H^2\dot\varphi^2 + V(\varphi), \tag{7}$$

$$P_\varphi = -\frac{\xi}{M^2}\left(3H^2\dot\varphi^2 + 2\dot H\dot\varphi^2 + 4H\dot\varphi\ddot\varphi\right) - V(\varphi), \tag{8}$$

[1] We adopt the notation of sign difference as $(-,+,+,+)$.

respectively. Moreover, the equation of motion (EoM) of φ can be written as:

$$\frac{6\xi}{M^2}H^2\ddot{\varphi}+\frac{6\xi}{M^2}\left(2\dot{H}+3H^2\right)H\dot{\varphi}+V_{\varphi}=0. \tag{9}$$

It's also useful to define parameters like:

$$y\equiv\frac{\xi}{M^2}\dot{\varphi}^2,\qquad \eta\equiv\frac{\ddot{\varphi}}{H\dot{\varphi}},\qquad \epsilon\equiv-\frac{\dot{H}}{H^2},\qquad \epsilon_{\phi}\equiv\frac{\dot{\phi}^2}{M_p^2H^2}, \tag{10}$$

which we will use later. One can also notice that $\dot{y}=2\frac{\xi}{M^2}\dot{\varphi}\ddot{\varphi}=2Hy\eta$.

As a general study, in the following we will briefly review the evolution behavior of the curvaton field φ according to Eqs. (7)–(9). We will divide the whole analysis into three classes:

1) $V(\varphi)=0$. In this class, the energy density of ρ_{φ} is contributed only by its kinetic term, and the EoM (9) can be easily solved without V_{φ}. According to the solution, $\dot{\varphi}$ scales as $a^{-3}H^{-2}$, then the energy density scales as $\rho_{\varphi}\sim H^2\dot{\varphi}^2\sim a^{-6}H^{-2}$. The scaling behavior of a and H is determined by the background energy density $\rho_{bg}\propto a^{-3(1+w)}$, so one can straightforwardly have $\rho_{bg}\sim$ constant, $\rho_{\varphi}\propto a^{-6}$ for $w=-1$, $\rho_{bg}\propto a^{-3}$, $\rho_{\varphi}\propto a^{-3}$ for $w=0$, $\rho_{bg}\propto a^{-4}$, $\rho_{\varphi}\propto a^{-2}$ for $w=1/3$, and $\rho_{bg}\propto a^{-6}$, $\rho_{\varphi}\sim$ constant for $w=1$. One can see that this forms a duality between the energy densities of the background and φ, the relation of which is $\rho_{\varphi}\rho_{bg}\sim a^{-6}\sim\rho_m^2$, where ρ_m denotes energy density of nonrelativistic matter. This is an interesting property of the nonminimal derivative coupling models subdominant in the universe. Similar arguments have also been made in [39].

2) $V(\varphi)\neq0$, $V_{\varphi}=0$. In this case, the EoM of φ is the same as in the above case, so φ has the same solution as $\dot{\varphi}\sim a^{-3}H^{-2}$, however ρ_{φ} will be added by a constant potential $V=V_0$. For power-law ansatz solution, $a(t)\sim t^{2/3(1+w)}$, one can get the scaling of the kinetic term of ρ_{φ}, $\xi H^2\dot{\varphi}^2\sim a^{3(w-1)}$. For different background where w and scaling of a are different, this term can be increasing or decreasing, or remains constant, and ρ_{φ} can be dominated by either kinetic term or potential. In the expanding universe, when $w>1$ the kinetic term is increasing and will dominate ρ_{φ}, and when $w<1$ it is decreasing and ρ_{φ} will be dominated by the potential. Things become vice versa in contracting universe, and in both cases, the two part will scale in-phase and contribute to ρ_{φ} together for $w=1$.

3) $V(\varphi)\neq0$, $V_{\varphi}\neq0$. This is the most general and complicated case and usually the exact solution cannot be obtained analytically. Even though, for some simple cases, we can still find some ansatz solutions. One of the examples is the scaling solution, of which all the terms in EoM and Friedmann equations have the same scaling w.r.t t. Assuming $\varphi\sim t^c$ where c is a constant parameter, one finds that to have the terms in EoM have the same scaling requires $V_{\varphi}\sim t^{c-4}$, which in turn gives $V(\varphi)\sim\varphi^{2-4/c}$. Thus one has $\rho_{\varphi}\sim t^{2c-4}$, while $\rho_{bg}\sim H^2\sim t^{-2}$. For $c<1$, when $|t|\to\infty$ (late time or early time in bouncing cosmology), ρ_{φ} will not affect the background much, but will have significant effect at $|t|\to0$ (approaching to big-bang singularity). Vice versa for $c>1$. Note that for $c=1$ where $\dot{\varphi}$ becomes constant, the energy density of φ will have the same scaling as that of the background.

4. Perturbation

4.1. Scalar perturbation

In Section 2, we demonstrated that this kind of curvaton model can give rise to scale-invariant power spectrum without knowing the exact behavior of the universe, which is due to the nonminimal derivative coupling. There is, though, a small stumbling block in front of us before we cheer for such an easy but interesting expectation. As we introduce the nonminimal coupling of curvaton to gravity, the gravitational perturbations, which have always been neglected for curvaton models, might get invoked, although one of the components can be gauged away. Such an effect may or may not change the perturbed action (2) substantially, which is the basis of our result, so one should be careful in treating perturbations of such a model and consider the gravitational perturbations as well. So in this section, we will analyze the perturbations of this model with careful calculation. We will show that, fortunately, the gravitational perturbations will indeed not play an important role in our model, and our expected result still holds very well.

The perturbed metric can be written as follows:

$$ds^2=-N^2dt^2+h_{ij}\left(dx^i+N^idt\right)\left(dx^j+N^jdt\right), \tag{11}$$

where N is the lapse function, N^i is the shift vector, and h_{ij} is the induced 3-metric. One can then perturb these functions as:

$$N=1+\alpha,\qquad N_i=\partial_i\beta,\qquad h_{ij}=a^2(t)e^{2\psi}\delta_{ij}, \tag{12}$$

where α, β and ψ are the scalar metric perturbations. As for curvaton models, it is convenient to consider the spatial-flat gauge, where $\psi=0$. Moreover, the perturbation of φ field is

$$\varphi\to\varphi(t)+\delta\varphi(t,\mathbf{x}). \tag{13}$$

The perturbation generated by the background field ϕ is also neglected.

Using this, we can expand our action (6) up to the second order. The expansion and reduction processes are rather tedious, as can be seen for the nonminimal derivative coupling term in Appendix A, after which we can obtain the constraining degrees of freedom α and β, which appear to be:

$$\alpha=a_1\dot{\delta\varphi}+a_2\delta\varphi,\qquad \partial^2\beta=b_1\dot{\delta\varphi}+b_2\delta\varphi+b_3\partial^2\delta\varphi, \tag{14}$$

where we define

$$a_1\equiv-\frac{2\xi\dot{\varphi}/M^2}{M_p^2-3y},\qquad a_2\equiv\frac{3\xi H\dot{\varphi}/M^2}{M_p^2-3y}, \tag{15}$$

$$b_1\equiv a^2\frac{[a_1\frac{\dot{\phi}^2}{2H}-\frac{9\xi H\dot{\varphi}}{M^2}-3Ha_1(M_p^2-6y)]}{M_p^2-3y}, \tag{16}$$

$$b_2\equiv a^2\frac{[a_2\frac{\dot{\phi}^2}{2H}-3Ha_2(M_p^2-6y)-\frac{V_{,\varphi}}{2H}]}{M_p^2-3y}, \tag{17}$$

$$b_3\equiv-\frac{2\xi\dot{\varphi}/M^2}{M_p^2-3y}. \tag{18}$$

Substituting this into (6), and use conformal time η instead of cosmic time t, one will finally find that the second order perturbation action appears as:

$$\delta S_{\varphi}=\int d\eta\, d^3xa^2\frac{Q}{c_s^2}\left[\delta\varphi'^2-c_s^2\partial_i\delta\varphi\partial^i\delta\varphi-\frac{1}{2}\frac{a^2c_s^2m_{eff}^2}{Q}\delta\varphi^2\right], \tag{19}$$

where

$$Q\equiv-\frac{\xi H^2}{M^2}\left[\frac{4y(4-\epsilon+2\eta)}{M_p^2-3y}+\frac{24y^2\eta}{(M_p^2-3y)^2}+2\epsilon-7\right],$$

$$c_s^2\equiv\frac{QM^2}{\xi H^2}\left[2y\frac{(12+\epsilon_{\phi})M_p^2-18y}{(M_p^2-3y)^2}+3\right]^{-1},$$

$$m_{eff}^2=\frac{\xi}{M^2}\frac{3M_p^2H^4y}{(M_p^2-3y)^2}\left[3\left(6\epsilon-3\epsilon_{\phi}-4\eta\frac{M_p^2+3y}{M_p^2-3y}\right)\right.$$

$$-2\epsilon_\phi\left(\frac{\dot{\epsilon}_\phi}{H\epsilon_\phi}-3\epsilon+2\eta\frac{M_p^2+3y}{M_p^2-3y}\right)\Bigg]+\frac{M_p^2-y}{M_p^2-3y}V_{\varphi\varphi}$$
$$-54\frac{\xi}{M^2}\frac{H^4y^2}{(M_p^2+3y)^2}\left(9-3\epsilon+4\eta\frac{M_p^2}{M_p^2-3y}\right)$$
$$-12\frac{\xi}{M^2}\frac{H^4y}{M_p^2-3y}\left(6+\eta\frac{M_p^2+3y}{M_p^2-3y}\right)(\eta-2\epsilon+3). \quad (20)$$

Note that in the above formulation, we have made use of the background equation of motion (9) as well as the definitions of the parameters in Eq. (10).

Define $z\equiv a\sqrt{Q}/c_s$, we can get the equation of motion of $\delta\varphi$ in its neat form as:

$$(z\delta\varphi)''+\left(c_s^2k^2-\frac{z''}{z}+\frac{1}{2}\frac{a^4m_{eff}^2}{z^2}\right)(z\delta\varphi)=0, \quad (21)$$

and as will be shown explicitly later, for the case of $Q\sim H^2$ and negligible m_{eff}^2, we have the approximate solution

$$\delta\varphi=k^\nu(\eta_*-\eta)^{\nu+\frac{3}{2}},\quad k^{-\nu}(\eta_*-\eta)^{\nu+\frac{3}{2}},\ \nu\simeq\frac{3}{2}, \quad (22)$$

where we've made use of $z^2\sim a^2H^2\sim(\eta_*-\eta)^{-2}$. From this solution we can see that, whether expanding or contracting the universe will be, the last mode (which is constant) will dominate over the first one (which is decaying), and gives rise to scale-invariant power spectrum. This is because the factor Q has some "faking" effect, making the perturbation $\delta\varphi$ "feel" itself in a de-Sitter expanding phase, even though the real evolution is not. This is an interesting application of the "conformal mechanism" in [16–18] and is the essential property of this model that we are pursuing. Moreover, the power spectrum of $\delta\varphi$, $\mathcal{P}_{\delta\varphi}$, and the spectral index, n_s, are defined as:

$$\mathcal{P}_{\delta\varphi}\equiv\frac{k^3}{2\pi^2}\frac{|\delta\varphi|^2}{M_p^2},\qquad n_s-1\equiv\frac{d\ln\mathcal{P}}{d\ln k}, \quad (23)$$

respectively.

To have analytical solutions we take two interesting limits. The first case is that $|y|\ll M_p^2$, meaning that the velocity of the curvaton field is quite slow and the kinetic term of curvaton is negligible. In this case, we have:

$$Q\simeq-\frac{\xi H^2}{M^2}(2\epsilon-7),\qquad c_s^2\simeq-\frac{1}{3}(2\epsilon-7),\qquad m_{eff}^2\simeq V_{\varphi\varphi}. \quad (24)$$

When $V(\varphi)$ is sufficiently flat, $V_{\varphi\varphi}\ll H^2$, the effective mass term can also be neglected. In order to make this model free of ghost and gradient instabilities, we require $Q>0$, $c_s^2>0$, which leads to

$$\xi>0,\qquad w=-1+\frac{2}{3}\epsilon<\frac{4}{3}, \quad (25)$$

which is the region of viability of our model in this case.

One can get the power spectrum of $\delta\varphi$ from Eqs. (22) and (23), which is:

$$\mathcal{P}_{\delta\varphi}=\frac{H^2}{4\pi^2M_p^2c_s^3Q}=\sqrt{\frac{27}{(7-2\epsilon)^5}}\frac{M^2}{4\pi^2M_p^2\xi}, \quad (26)$$

where in the last step, we've made use of the results in Eq. (24). Note also that our result indicates that the spectrum of $\delta\varphi$ is determined by the cutoff scale M instead of H, and H will not be constrained by the spectrum. However, as will be seen very soon, the dependence on H will be turned on when the perturbations of curvaton field transfer into the adiabatic perturbations.

In this result, the power spectrum is exactly scale-invariant if there is no other correction term, however, the observational data from PLANCK [14] favors small tilt in the power spectrum index. Recall that we have turned off the mass term for simplicity, and when this term is turned on, we may get this small correction. Considering mass term from (24), the equation of motion becomes:

$$(z\delta\varphi)''+\left(c_s^2k^2-\frac{z''}{z}+\frac{1}{2}\frac{a^4V_{\varphi\varphi}}{z^2}\right)(z\delta\varphi)=0. \quad (27)$$

For a specific choice, we choose $V_{\varphi\varphi}\sim H^4\sim t^{-4}$, which can be reconstructed to give $V(\phi)\sim t^{2c-4}\sim\varphi^{2-4/c}$, according to our background analysis in Section 2. This ansatz gives the scaling of the last term w.r.t. the conformal time, $a^4V_{\varphi\varphi}/2z^2\propto(\eta_*-\eta)^{-2}$. Setting the prefactor to be Δ_1, and moreover since $z''/z\sim2/(\eta_*-\eta)^2$, Eq. (27) becomes

$$(z\delta\varphi)''+\left[c_s^2k^2-\frac{2-\Delta_1}{(\eta_*-\eta)^2}\right](z\delta\varphi)=0, \quad (28)$$

which one can solve to get the spectral index n_s (defined through $n_s\equiv d\ln P/d\ln k$) of our model in this case:

$$n_s-1=\frac{2}{3}\Delta_1. \quad (29)$$

The second case of the analytical solution of our model is that $|y|\gg M_p^2$, indicating that the curvaton moves with a very large speed. In realistic models, this case is somehow dangerous, since this might give a large kinetic term to curvaton, and to make the energy density of the curvaton field exceed that of the background, one might need also a large potential term with opposite sign to cancel the kinetic energy, which requires fine-tuning in some level, so one may need to be careful to make it a healthy model. However here as a complete study, we will also consider this case. From Eq. (20) we have:

$$Q\simeq-\frac{\xi H^2}{3M^2}(10\epsilon-37),\qquad c_s^2\simeq\frac{1}{3}(10\epsilon-37),$$
$$m_{eff}^2\simeq-2\frac{\xi}{M^2}H^4\left(9+15\epsilon-6\eta+2\eta^2-4\eta\epsilon\right)+\frac{1}{3}V_{\varphi\varphi}, \quad (30)$$

and the positivity of Q and c_s^2 requires

$$\xi<0,\qquad w>\frac{22}{15}. \quad (31)$$

Differently from the previous case, here we obtained a correction that is proportional to H^4 which is brought by considering metric perturbation. Similarly, one can also obtain corrections in the equation of motion of $\delta\varphi$, which is proportional to $(\eta_*-\eta)^{-2}$. Setting the prefactor to be Δ_2, one can get the power spectrum and its index as:

$$\mathcal{P}_{\delta\varphi}=\frac{H^2}{4\pi^2M_p^2c_s^3Q}$$
$$=\sqrt{\frac{243}{(10\epsilon-37)^5}}\frac{M^2}{4\pi^2M_p^2(-\xi)}, \quad (32)$$
$$n_s-1=\frac{2}{3}\Delta_2. \quad (33)$$

As a side remark, we should mention that a non-vanishing $V_{\varphi\varphi}$ can also provide tilt of the spectrum as well. However, as can be seen in the next section, when we consider the tensor perturbations of this model, this case will be ruled out since it induces the instability of tensor perturbations by having the sound speed of gravitational waves $c_T^2<0$.

4.2. Tensor perturbation

The recently release PLANCK data not only did a well measurement for scalar perturbations in the early universe, but also plans to measure the tensor perturbations, i.e. the gravitational waves. Whatever the future result is, one conclusion that can now be confirmed is that the signature of gravitational waves is quite small. This can already have some constraints on theoretical models. To make our analysis complete, we also consider the tensor perturbations of our model.[2] The metric containing tensor perturbation is written as:

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + \gamma_{ij})\,dx^i\,dx^j, \tag{34}$$

where $\gamma_{ij}(x)$ is the tensor perturbation satisfying traceless and transverse conditions:

$$\gamma_i^i = 0, \qquad \partial_i \gamma_j^i = 0. \tag{35}$$

Noting that in our model, the field part also has contributions to tensor perturbations due to the nonminimal coupling, we can perturb the action (6) up to second order as:

$$\delta S_\varphi^T = \frac{1}{8}\int dt\,d^3x\,a^3\left[\mathcal{F}\dot{\gamma}_{ij}^2 - \mathcal{G}\frac{(\nabla\gamma_{ij})^2}{a^2}\right], \tag{36}$$

where we defined

$$\mathcal{F} = M_p^2 - y, \qquad \mathcal{G} = M_p^2 + y, \tag{37}$$

where y is defined as in (10). Note that this action is consistent with that of generalized Galileon action in [44]. From this action, one can read that the squared sound speed of gravitational waves is:

$$c_T^2 \equiv \frac{\mathcal{G}}{\mathcal{F}} = \frac{M_p^2 + y}{M_p^2 - y}. \tag{38}$$

In the previous analysis for scalar perturbation, we discussed about two limited cases, namely $|y| \ll M_p^2$ and $|y| \gg M_p^2$. However, from the above expression of c_T^2 we can see that, the last case will induce $c_T^2 \simeq -1 < 0$, which will induce an unstable tensor perturbation. So this case should be abandoned, leaving only the first case, which gives $c_T^2 \simeq 1$, namely a healthy tensor perturbation. Therefore in the following, we will only consider this case. In this case, we have $\mathcal{F} \simeq M_p^2$, $\mathcal{G} \simeq M_p^2$, and thus the behavior of the tensor perturbation will actually be very close to that in the case of minimal coupling single scalar, in which the primordial tensor perturbation spectra from various expanding and contracting phases have been calculated in [43]. Taking the conditions of constant w, one can derive the equation of motion for γ_{ij} from Eq. (36) and get the solution:

$$\gamma_{ij} = \text{constant}, \qquad \int \frac{dt}{a^3(t)M_p^2}, \tag{39}$$

where $a(t)$ can be parametrized as $a(t) \sim t^{2/3(1+w)}$. From this result we can easily see that the tensor perturbation is dominated by its constant mode when $w > 1$ for contracting phase or $w > -1/3$ for expanding phase where the varying mode is actually decreasing, while by its varying mode when $-1/3 < w < 1$ for the contracting phase where the varying mode is actually growing.[3] By a detailed calculation, the tensor spectrum is obtained as:

$$\mathcal{P}_T \sim k^3|\gamma_{ij}|^2 \sim \frac{H^2}{M_p^2}\left(\frac{k}{k_0}\right)^{n_T}, \tag{40}$$

where the spectral index

$$n_T = \frac{6(1+w)}{1+3w} \quad (w > 1), \quad \text{or} \quad \frac{12w}{1+3w} \quad \left(-\frac{1}{3} < w < 1\right) \tag{41}$$

for contracting phase and

$$n_T = \frac{6(1+w)}{1+3w} \quad \left(w < -\frac{1}{3}\right) \tag{42}$$

for expanding phase. Here k_0 denotes some pivot wavenumber.

One can see from the result that, in order not to get too much gravitational waves, we need either scale-invariant tensor spectrum with the amplitude $|\mathcal{P}_T|^{1/2} \sim M_p^{-1}H \sim 10^{-5}$, or blue-tilted tensor spectrum of which the amplitude can be large (namely, H can be larger than $10^{-5}M_p$), which could be suppressed by power-laws of k on large scales (cf. [11]). This requires the background equation of state be either no less than 0 (for contracting phase) or no more than -1 (for expanding phase).[4] Combining the constraints on w that were obtained previously for scalar perturbations, we can conclude that the viable condition for our model is that

$$\begin{cases} 0 < w < \frac{4}{3} & \text{for contracting phase,} \\ w < -1 & \text{for expanding phase,} \end{cases} \tag{43}$$

which is another important conclusion of our model.

5. The creation of curvature perturbation and Local-type non-Gaussianity

In the above section, we showed that our model is able to give rise to scale-invariant perturbations with large range of universe evolution, provided constraints (25) and (31) hold in order to keep the perturbation of the model well-defined. However, these perturbations are isocurvature ones. The adiabatic curvature perturbation, which can be observable, can be obtained in two ways: one is when the background decays and the curvaton dominates the universe, and the other is when the curvaton and background decay simultaneously and their decay products become equilibrium. Here we assume that the background decays into relativistic matter for simplicity. The final curvature perturbation can be expressed as:

$$\zeta = \frac{\delta\rho_\varphi}{4\rho_r + 3(\rho_\varphi + P_\varphi)}, \tag{44}$$

where the density perturbation $\delta\rho_\varphi$ can further be expanded with respect to $\delta\varphi$. The linear and next-to-linear order of $\delta\rho_\varphi$ is given by:

$$\delta^{(1)}\rho_\varphi \equiv \rho_{\varphi,\varphi}\delta\varphi \simeq \left(-\frac{18\xi\epsilon H^3\dot{\varphi}}{M^2} + V_{,\varphi}\right)\delta\varphi, \tag{45}$$

$$\delta^{(2)}\rho_\varphi \equiv \frac{1}{2}\rho_{\varphi,\varphi\varphi}\delta\varphi^2 \simeq \frac{1}{2}\left(\frac{54\xi\epsilon^2H^4}{M^2} + V_{,\varphi\varphi}\right)\delta\varphi^2, \tag{46}$$

respectively.

[2] We thank the referee for pointing us this issue.

[3] The boundary of $-1/3$ is set for the requirement of avoidance of the horizon problem.

[4] We do not mean that w's out of this range are completely ruled out, since this is only a theoretical estimation. For example, as in usual inflation case, although $w > -1$ causes a red-tilted tensor spectrum which will be raised on large scales, it can still survive if the deviation of w from -1 is not too much (namely under slow-roll condition), such that the raising effect does not conflict with the observational data.

Now we can consider the two ways separately. If the curvaton dominates the universe, say $\rho_\varphi \gg \rho_r$, from Eq. (44) we have:

$$\zeta^A \simeq \frac{\delta^{(1)}\rho_\varphi}{3(\rho_\varphi + P_\varphi)} \simeq \frac{\rho_{\varphi,\varphi}}{3(\rho_\varphi + P_\varphi)}\delta\varphi \simeq -\frac{3\epsilon H_*}{\dot\varphi_*(3+\epsilon-2\eta)}\delta\varphi, \tag{47}$$

where H_* and $\dot\varphi_*$ are the values of H and $\dot\varphi$ at the corresponding time, while if the curvaton decays before its dominance, it will only contribute part of the energy density of the universe. Defining $r \equiv \rho_\varphi/\rho_r$, one has:

$$\zeta^B \simeq \frac{r}{4}\frac{\delta^{(1)}\rho_\varphi}{\rho_\varphi} \simeq \frac{r}{4}\frac{\rho_{\varphi,\varphi}}{\rho_\varphi}\delta\varphi = -\frac{\epsilon r H_*}{2\dot\varphi_*}\delta\varphi, \tag{48}$$

where the supscripts A and B refer to the two cases respectively. In both cases, we neglected contribution from potential term which is subdominant.

The power spectrum of curvature perturbation is defined as $k^3|\zeta|^2/2\pi^2$ in usual convention. Following results from Eqs. (47) and (48), one easily gets that for the case $|y| \ll M_p^2$:

$$\mathcal{P}_\zeta^A \simeq \frac{27\sqrt{3}H_*^2}{4\pi^2|y_*|(7-2\epsilon)^{5/2}}\left(\frac{\epsilon}{3+\epsilon-2\eta}\right)^2, \tag{49}$$

$$\mathcal{P}_\zeta^B \simeq \frac{3\sqrt{3}\epsilon^2 r^2 H_*^2}{16\pi^2|y_*|(7-2\epsilon)^{5/2}}, \tag{50}$$

where y_* is the value of y at the corresponding time. The observational data constrains the amplitude of the power spectrum as $\mathcal{P}_\zeta = (2.23 \pm 0.16)\times 10^{-9}$ (68% C.L.), and in usual inflation/curvaton models, to be consistent with this constraint one needs roughly $H \simeq 10^{-5}M_p$. In our results we can see, for typical values of parameters $\epsilon, \eta, r \sim \mathcal{O}(1)$, we have $\mathcal{P}_\zeta \sim H_*^2/|y_*|$, so when $|y| \ll M_p^2$, we may get a lower scale inflation with $H \ll 10^{-5}M_p$.

Moreover, as one can see from the derivation, the spectral index of the power spectrum of ζ can be directly inherited from that of $\delta\varphi$, namely Eqs. (29) and (33), without correction during the transfer. Therefore, in order to make our model consistent with observation result $n_s \simeq 0.96$ by PLANCK data [14], one should constrain Δ_1 to be close to about -0.06.

In recent years, especially after the release of PLANCK data, the non-Gaussianities of primordial perturbations become more and more hot in the studies of the early universe. This is not only due to the great degeneracy in power spectrum of the early universe models, but also because of the more and more accurate measurements of the nonlinear perturbations. In the following of this section, we will focus on the non-Gaussianities generated by our model.

As a curvaton model where the adiabatic perturbations are generated at superhubble scale, the non-Gaussianities are mostly of local type. The local type non-Gaussianities of curvature perturbation are given by:

$$\zeta = \zeta_g + \frac{3}{5}f_{NL}^{local}\zeta_g^2, \tag{51}$$

where the subscript "g" denotes the Gaussian part of ζ while f_{NL} is the so-called nonlinear estimator. For local type, f_{NL}^{local} can be estimated by using the so-called δN [45]:

$$\zeta = \delta N = N_{,\varphi}\delta\varphi + \frac{1}{2}N_{,\varphi\varphi}\delta\varphi^2 + \cdots, \tag{52}$$

where $N \equiv \ln a$. Comparing Eqs. (51) and (52) one can easily find the relation:

$$f_{NL}^{local}\big|_\zeta = \frac{5}{6}\frac{N_{,\varphi\varphi}}{N_{,\varphi}^2}, \tag{53}$$

and from Eq. (44), we have:

$$N_{,\varphi} = \frac{\rho_{\varphi,\varphi}}{4\rho_r + 3(\rho_\varphi + P_\varphi)}, \qquad N_{,\varphi\varphi} = \frac{\rho_{\varphi,\varphi\varphi}}{4\rho_r + 3(\rho_\varphi + P_\varphi)}, \tag{54}$$

respectively.

Now we consider the two cases separately. For the first case where curvaton dominates the energy density before decays, one gets:

$$f_{NL}^{local}\big|_\zeta^A \simeq \frac{5}{2}\frac{(\rho_\varphi + P_\varphi)\rho_{\varphi,\varphi\varphi}}{\rho_{\varphi,\varphi}^2} \simeq \frac{5}{6}(3+\epsilon-2\eta), \tag{55}$$

and for the second case where the curvaton decays and never dominates the energy density, we have

$$f_{NL}^{local}\big|_\zeta^B \simeq \frac{10}{3r}\frac{\rho_\varphi\rho_{\varphi,\varphi\varphi}}{\rho_{\varphi,\varphi}^2} \simeq \frac{5}{r}, \tag{56}$$

respectively. In recent PLANCK paper [46], the local-type non-Gaussianities have been constrained as $f_{NL}^{local} = 2.7 \pm 5.8$ (68% C.L.). With reasonable choices of parameters ϵ, η and r to be roughly (or smaller than) $\mathcal{O}(1)$, one can see that the local-type non-Gaussianities of our model are well within the observational constraints by the PLANCK data.

6. Discussion

In this Letter, we studied a new kind of curvaton model with its kinetic term nonminimally coupled to the Einstein tensor. This kind of coupling will contribute a factor of H^2 to the kinetic term of curvaton. Various kinds of the background evolutions of the curvaton field are reviewed, and a complete analysis of the perturbation theory of the model, including corrections from gravitational perturbations, is performed. Thanks to such a coupling, the perturbations feel like in a nearly de-Sitter spacetime, which will give rise to scale-invariant power spectrum favored by the data, independent of the details of the background evolution of the universe. Although the analysis becomes complicated when gravitational perturbations are involved in, we showed that the conclusion still holds qualitatively in large-speed and small-speed limits. The small tilt of the power spectrum might be obtained by the corrections from the potential of the curvaton field. Taking into account the conditions that scalar and tensor perturbations are stable can impose some constraints on the background, but still a quite large range of background EOS could be allowed. Moreover, this simple model can also be generated local-type non-Gaussianities of $\mathcal{O}(1)$, which is favored by the recent PLANCK data.

As a natural extention, we note that if $|\epsilon| \gg 1$ is rapidly changed, the scale factor $a(\eta)$ might evolve as a constant [20]. From the last relation of Eq. (5), $Q \sim 1/t^2$ has to be satisfied. The evolution with $|\epsilon| \gg 1$ can be parameterized as $H \sim (t_* - t)^{-b}$, which leads to $Q \sim H^{2/b}$, thus the scale invariance requires $Q \sim (R/M^2)^{1/b}$. The kinetic term in such a case is more complicated for analytic calculation, and seems hard to be written in a covariant form as $G_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi$. Although coming from the same logic, this gives us an independent model, so we leave the discussion on such kind of models for future work.

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Appendix A. 3 + 1 decomposition

In deriving perturbed action (19) for actions that contain more general gravity terms such as $G_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi$, it is necessary to know how 3+1 decomposition can be done to such terms. This appendix is devoted to make clear how the 3 + 1 form of $G_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi$ can be obtained. We follow the perturbed metric shown in Eq. (11):

$$ds^2 = -N^2dt^2 + h_{ij}\big(dx^i + N^i dt\big)\big(dx^j + N^j dt\big). \tag{A.1}$$

First of all, it is useful to define Normal vector of the 3-dimensional hypersurface: $n_\mu = n_0(dt/dx^\mu) = (n_0, 0, 0, 0)$ and $n^\mu \equiv g^{\mu\nu}n_\nu$. Using the normalization $n_\mu n^\mu = -1$ one can determine $n_0 = N$, so

$$n_\mu = (N, 0, 0, 0), \qquad n^\mu = \left(-\frac{1}{N}, \frac{N^i}{N}\right), \tag{A.2}$$

and the 3-dimensional induced metric, $H_{\mu\nu}$, which is defined to be orthogonal to the normal vector ($H_{\mu\nu}n^\nu = 0$), can be chosen as

$$H_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu. \tag{A.3}$$

Moreover, the corresponding contravariant form can be defined as $H^{\mu\nu} = g^{\mu\nu} + n^\mu n^\nu$, with $H^{0\mu} = 0$.

From now on, one can express $G_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi$ using 3-metric:

$$\begin{aligned} G_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi &= G_{\mu\nu}\big(H^{\mu\alpha} - n^\mu n^\alpha\big)\big(H^{\nu\beta} - n^\nu n^\beta\big)\partial_\alpha\varphi\partial_\beta\varphi \\ &= G_{\mu\nu}H^{\mu\alpha}H^{\nu\beta}\partial_\alpha\varphi\partial_\beta\varphi \\ &\quad - 2\big(G_{\mu\nu}n^\nu H^{\mu\alpha}\big)\partial_\alpha\varphi\big(n^\beta\partial_\beta\varphi\big) \\ &\quad + \big(G_{\mu\nu}n^\mu n^\nu\big)\big(n^\alpha\partial_\alpha\varphi\big)\big(n^\beta\partial_\beta\varphi\big), \end{aligned} \tag{A.4}$$

however, we still need to express $G_{\mu\nu}H^{\mu\alpha}H^{\nu\beta}$, $G_{\mu\nu}n^\nu H^{\mu\alpha}$ and $G_{\mu\nu}n^\mu n^\nu$ with 1- or 3-dimensional elements in (11). As we will show below, their expressions are nothing but Gauss, Codazzi and Ricci equations, which should be familiar to most people who study General Relativity.

Let's first study some properties of the 3-metric, $H_{\mu\nu}$. Firstly, the covariant derivative w.r.t. induced metric $H_{\mu\nu}$ is defined as:

$$D_\mu\mathcal{T}_\nu^\rho \equiv H_\mu^{\mu'}H_{\rho'}^{\rho}H_\nu^{\nu'}\nabla_\mu\mathcal{T}_\nu^\rho, \tag{A.5}$$

where $\mathcal{T}_\nu^\rho$ is an arbitrary tensor, ∇ is the covariant derivative w.r.t. $g_{\mu\nu}$. One can check that $DH_{\mu\nu} = 0$. From the property of $H_{\mu\nu}$, one can also have $\overset{(H)}{\Gamma}{}^i_{jk} = \overset{(h)}{\Gamma}{}^i_{jk}$ where they are connections for $H_{\mu\nu}$ and h_{ij} respectively, so one has

$$D_iV_j = \tilde\nabla_iV_j, \tag{A.6}$$

where $\tilde\nabla$ is the covariant derivative w.r.t. h_{ij} ($\tilde\nabla h_{ij} = 0$.) Moreover, $\overset{(H)}{\Gamma}{}^0_{\mu\nu} = 0$ because of the fact that $H^{0\mu} = 0$.

Moreover, the curvature of 3-dimensional hypersurface is described by the extrinsic curvature $K_{\mu\nu}$, with the definition:

$$\begin{aligned} K_{\mu\nu} &\equiv \frac{1}{2}\mathcal{L}_nH_{\mu\nu} \\ &= \frac{1}{2N}\big(\dot H_{\mu\nu} - D_\mu N_\nu - D_\nu N_\mu\big) \end{aligned} \tag{A.7}$$

where $\mathcal{L}_n$ is the Lie derivative w.r.t. n^μ. Since $H_{ij} = h_{ij}$ and $D_iN_j = \tilde\nabla_iN_j$, we have

$$K_{ij} = \tilde K_{ij}, \tag{A.8}$$

the right hand side of which is defined as $K_{ij} = (\dot h_{ij} - \tilde\nabla_iN_j - \tilde\nabla_jN_i)/2N$. Furthermore, from the relation $K^{\mu\nu} = H^{\mu\mu'}H^{\nu\nu'}K_{\mu\nu}$ we have $K^{0\mu} = 0$ and $K^{ij} = \tilde K^{ij}$. Note that $K_{\mu\nu}$ can also be written as

$$K_{\mu\nu} = H_\mu^{\mu'}H_\nu^{\nu'}\nabla_{\mu'}n_{\nu'} = H_\mu^{\mu'}\nabla_{\mu'}n_\nu. \tag{A.9}$$

Therefore it is easy to check that $K_{\mu\nu}n^\mu = 0$.

The 3-dimensional induced Riemann tensor (induced means generated by $H_{\mu\nu}$, the same hereafter) is defined by:

$${}^{(3)}R^\sigma{}_{\mu\nu\rho} = H^\sigma_{\sigma'}H_\mu^{\mu'}H_\nu^{\nu'}H_\rho^{\rho'}R^\sigma{}_{\mu\nu\rho} - 2K_\nu{}^\sigma K_{\mu\rho} + 2K_{\nu\mu}K^\sigma{}_\rho, \tag{A.10}$$

where $R^\sigma{}_{\mu\nu\rho}$ is the 4-dimensional Riemann tensor (generated by $g_{\mu\nu}$). The indices of ${}^{(3)}R^\sigma{}_{\mu\nu\rho}$ are raised and lowered by $H_{\mu\nu}$. Moreover, it satisfies the relation:

$$(D_\mu D_\nu - D_\nu D_\mu)V^\sigma = {}^{(3)}R^\sigma{}_{\rho\mu\nu}V^\rho \tag{A.11}$$

for any spatial vector that satisfies $n_\sigma V^\sigma = 0$.

The contraction of ${}^{(3)}R^\sigma{}_{\mu\nu\rho}$ gives induced Ricci tensor ${}^{(3)}R_{\mu\nu}$, and from the definition $R_{\mu\nu} \equiv \Gamma^\alpha_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\alpha,\nu} + \Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\alpha\beta} - \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha}$ along with the condition $\overset{(H)}{\Gamma}{}^0_{\mu\nu} = 0$ and $\overset{(H)}{\Gamma}{}^i_{jk} = \overset{(h)}{\Gamma}{}^i_{jk}$, one can also find that

$${}^{(3)}R_{ij} = \tilde R_{ij}, \tag{A.12}$$

where $\tilde R_{ij}$ corresponds to h_{ij}. Of course by contraction we also have ${}^{(3)}R = \tilde R$.

From this definition of ${}^{(3)}R^\sigma{}_{\mu\nu\rho}$, one can get **the Gauss equation:**

$$\begin{aligned} G_{\mu\nu}n^\mu n^\nu &= \frac{1}{2}\big({}^{(3)}R - K_{\mu\nu}K^{\mu\nu} + K^2\big) \\ &= \frac{1}{2}\big({}^{(3)}\tilde R - \tilde K_{ij}\tilde K^{ij} + \tilde K^2\big), \end{aligned} \tag{A.13}$$

the Codazzi equation:

$$\begin{aligned} G_{\mu\nu}n^\nu H^{\mu\rho} &= H^{\rho\sigma}\big(D_\alpha K^\alpha_\sigma - D_\sigma K\big), \\ G_{\mu\nu}n^\nu H^{\mu 0} &= 0, \\ G_{\mu\nu}n^\nu H^{\mu i} &= H^{ij}\big(D_kK^k_j - D_jK\big) \\ &= h^{ij}\big(\tilde\nabla_k\tilde K^k_j - \partial_j\tilde K\big), \end{aligned} \tag{A.14}$$

and **the Ricci equation:**

$$\begin{aligned} G_{\mu\nu}H^{\mu\alpha}H^{\nu\beta} = H^{\alpha\gamma}H^{\beta\delta}\bigg(&\frac{1}{N}\mathcal{L}_mK_{\gamma\delta} - \frac{1}{N}D_\gamma D_\delta N + {}^{(3)}R_{\gamma\delta} \\ &+ KK_{\gamma\delta} - 2K_{\gamma\epsilon}K^\epsilon_\delta\bigg) \\ &- \frac{1}{2}g_{\mu\nu}H^{\mu\alpha}H^{\nu\beta}\big[{}^{(3)}R - K^2 + K_{\rho\sigma}K^{\rho\sigma} \\ &+ 2\nabla_\rho\big(Kn^\rho - n^\sigma\nabla_\sigma n^\rho\big)\big] \quad \big(m^\mu = Nn^\mu\big), \end{aligned} \tag{A.15}$$

$$G_{\mu\nu}H^{\mu 0}H^{\nu\beta} = G_{\mu\nu}H^{\mu\alpha}H^{\nu 0} = 0, \tag{A.16}$$

$$\begin{aligned} G_{\mu\nu}H^{\mu i}H^{\nu j} = {} & \frac{1}{N}\dot{\tilde{K}}_{ij} - \frac{1}{N}\left(N^k\tilde{\nabla}_k\tilde{K}_{ij} + \tilde{K}^{jk}\tilde{\nabla}_i N_k + \tilde{K}^{ik}\tilde{\nabla}_j N_k\right) \\ & - \frac{1}{N}\tilde{\nabla}_i(\partial_j N) + {}^{(3)}\tilde{R}_{ij} + \tilde{K}\tilde{K}_{ij} - 2\tilde{K}_{ik}\tilde{K}^k_j \\ & - \frac{1}{2}h^{ij}\left[{}^{(3)}\tilde{R} - \tilde{K}^2 + \tilde{K}_{kl}\tilde{K}^{kl} + \frac{2}{N\sqrt{h}}\partial_0(\sqrt{h}\tilde{K}) \right. \\ & \left. - \frac{2}{N\sqrt{h}}\partial_k\left(\sqrt{h}N^k\tilde{K} + \sqrt{h}\partial^k N\right)\right]. \end{aligned} \tag{A.17}$$

These three equations show the $(3+1)$-form of the Einstein tensor (or equivalently, Ricci tensor). Moreover, by contraction we have:

$$\begin{aligned} R &= {}^{(3)}R - K^2 + K_{\mu\nu}K^{\mu\nu} + 2\nabla_\mu\left(Kn^\mu - n^\nu\nabla_\nu n^\mu\right) \\ &= {}^{(3)}\tilde{R} - \tilde{K}^2 + \tilde{K}_{ij}\tilde{K}^{ij} + \frac{2}{N\sqrt{h}}\partial_0(\sqrt{h}\tilde{K}) \\ &\quad - \frac{2}{N\sqrt{h}}\partial_i\left(\sqrt{h}N^i\partial_i\tilde{K} + \sqrt{h}\partial^i N\right) \end{aligned} \tag{A.18}$$

for Ricci scalar. Till now, all the needed variables of Gravity part have been decomposed and presented in terms of N, N_i and h_{ij}-related variables, which become computable. We refer the readers to [47] for more complete arguments.

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